Version 1.8 November 2011

GUIDE TO THE CALCULATION METHODS OF THE

FTSE ACTUARIES UK GILTS INDEX SERIES

FTSE

THE INDEX COMPANY

TABLE OF CONTENTS

SECTION 1

1.0 INTRODUCTION

1.1 Scope

This guide describes the scope of the FTSE Actuaries UK Gilts Index Series and the calculation methods used in their construction. It complements the document entitled "Ground Rules for the Management of the FTSE Actuaries UK Gilts Index Series" (Ground Rules). If a situation arises where this Guide and the Ground Rules can be interpreted differently, then the Ground Rules take priority.

The FTSE Actuaries UK Gilts Index Series cover separate calculations for conventional and index-linked gilts.

The aims of this Guide are:

(a) To describe how the gilt indices and their associated statistics are calculated;

(b) To make it easier for users to replicate the indices in order to support their trading and investment strategies;

(c) To assist users in understanding the component factors which influence the performance of the indices.

The guiding principles behind the calculation methods are:

(a) The calculation methods should reflect reality wherever practical;

(b) The indices should be capable of being replicated by users;

(c) Only historic data should be used in calculating the index statistics;

(d) Continuity with the past should be maintained wherever possible;

(e) The indices should be transparent and predictable;

(f) The primary purpose of the indices has always been to indicate the current level of yields in the market. A secondary purpose is to reflect accurately movements in the underlying market.

In order to replicate the indices, it is assumed that the investor is able to deal at closing middle market prices, without any expenses, and in any quantity. In addition, for the total return indices, it is assumed that the investor is able to reinvest the full interest amount, on the ex-dividend day, without any tax or expense considerations.

1.2 Management of the Indices

The FTSE Actuaries UK Gilts Index Series are calculated daily in accordance with the Ground Rules for the Management of the indices.

The FTSE Bond Indices Committee has been established jointly by FTSE International and the Faculty and Institute of Actuaries. The Committee may approve changes to the Ground Rules.

FTSE is responsible for the operation of the FTSE Actuaries UK Gilts Index Series, including the daily calculation of all the index values in accordance with the Ground Rules. FTSE maintains records of all the constituents and is responsible for the addition and deletion of gilts and changes in nominal amounts.

1.3 Overview of the Calculations

Due to their different structures, different but similar calculations are performed for the two main categories of gilts: conventional and index-linked securities. Each category is sub-divided into a number of sectors. The sectors are based on the outstanding terms to the assumed redemption dates of the gilts.

Conventional Gilts

The calculations for conventional gilts are divided into the following sectors according to their assumed redemption (maturity) dates:

All stocks – Gilts with all outstanding terms
Gilts with an assumed outstanding term of up to 5 years
Gilts with an assumed outstanding term of over 5 years
Gilts with an assumed outstanding term of 5 -10 years
Gilts with an assumed outstanding term of 10 -15 years
Gilts with an assumed outstanding term of 5 -15 years
Gilts with an assumed outstanding term of up to 15 years
Gilts with an assumed outstanding term of over 15 years
Gilts with an assumed outstanding term of up to 20 years
Gilts with an assumed outstanding term of 15 -25 years
Gilts with an assumed outstanding term of over 25 years
Irredeemable gilts – undated gilts

In addition fitted yields are calculated for the following terms to maturity:

5, 10, 15, 20, 25, 30, 35, 40, 45, 50 years and for irredeemables (undated gilts).

Index-Linked Gilts

For index-linked gilts, all of which have a single redemption date, price and coupon, indices are calculated for the following sectors:

All stocks – gilts with all outstanding terms
Gilts with an outstanding term of up to 5 years
Gilts with an outstanding term of 5 -15 years
Gilts with an outstanding term of 5 -25 years
Gilts with an outstanding term of over 5 years
Gilts with an outstanding term of over 15 years
Gilts with an outstanding term of 15 -25 years
Gilts with an outstanding term of over 25 years

There are no undated index-linked gilts.

The yields for the sectors are calculated assuming future inflation rates of 0%, 3%, 5% and 10%.

1.4 **Additional Details for Gilts Maturity Sector Descriptions**

The following details apply to both conventional and index-linked Gilts.

A maturity sector described in this Guide to Calculations as "up to XX years", is effectively defined and treated in the FTSE system as "up to, *but not including*, XX years".

A maturity sector described in this Guide to Calculations as "over XX years", is effectively defined and treated in the FTSE system as "*XX years and over*".

A maturity sector described in this Guide to Calculations as a range of "XX years to YY years", is effectively defined and treated in the FTSE system as "*including XX years and up to, but not including, YY years*".

These definitions will become important when we consider "shorteners" (covered in Rule 3.4.2) in the context of Gilts with *exactly* XX number of years to maturity. This is also necessary when clarifying FTSE's treatment of shorteners when they occur on a non-business day (i.e. weekend or holiday).

Dates

There are two relevant dates on each day: the "Calculation date", which is the date, after the close of which, the calculations are done, and the "Settlement date", which is the date when the deals are settled. Normally the settlement date is the next following working day after the calculation date, e.g. normally Tuesday after a Monday, etc, and Monday after a Friday, but with exceptions at public holidays. The calculation date is used to determine the outstanding term, and the settlement date is used for the calculation of accrued interest, redemption yields etc.

In this Guide "today" is used to mean the current Calculation date, "yesterday" to mean the previous Calculation date, and "tomorrow" to mean the next Calculation date.

All calculations of a Gilt's term to maturity are determined from Calculation Date rather than Settlement Date.

1.5 **Range of Calculations**

The main purpose of the indices is to calculate the level of yields in the market for Gilts of different outstanding terms[1]. A secondary purpose is to calculate price indices for the different sectors of the Gilts market. These price indices are supported by a range of other associated statistics. Different calculations are performed for conventional gilts and for index-linked gilts.

1. See the paper introducing the indices by G. M. Dobbie and A. D. Wilkie (Journal of the Institute of Actuaries Volume 105 Part 1 1978).

SECTION 1

Conventional Gilts

For each of the conventional sector indices the following information is calculated:

Gross (or dirty) price index *[2];
Accrued interest;
XD adjustment for the year to date;
Total return price index *;
Number of gilts in the index*;
Gross redemption yield;
Macaulay duration;
Modified duration;
Convexity;
Weight of the sector as a percentage of the total market
Day's Change*
Month's Change*
Year's Change*

In addition, yield indices are calculated which give the redemption yields for conventional gilts with outstanding terms of 5*, 10*, 15*, 20* years and for undated securities *.

2. Values marked with an asterisk * are displayed on a daily basis in the Financial Times.

Index-Linked Gilts

For each of the index-linked sector indices the following information is calculated:

Gross (or dirty) price index *;
Accrued interest;
XD adjustment for the year to date;
Total return price index *;
Number of gilts in the index;
Weight of the sector as a percentage of the total market *
Day's Change*
Month's Change*
Year's Change*

In addition for each sector, based on assumed future annual inflation rates of 0%*, 3%, 5%* and 10% of the UK Retail Price Index, the following information is calculated:

"Real" redemption yield *;
Macaulay duration;
Modified duration;
Convexity;

The "real" redemption yield of an index-linked gilt is its calculated gross redemption yield, after grossing up the scheduled future payments at the assumed rate of inflation, and then discounting the resulting value by the same assumed rate of inflation. In other words it produces a return in excess of the assumed inflation rate. It will be seen in the calculations, which are described in section 6, that this return is itself dependent on the assumed inflation rate.

The durations and convexity calculations above are based on the real yield calculations and hence are also dependent on the assumed inflation rate.

1.6 Publication

The indices are calculated at the end of each business day, and products are produced by FTSE on a subscription basis. Some information is also published in the *Financial Times* and is available on the FTSE web site www.ftse.com.

SECTION 2

2.0 GILTS INCLUDED IN THE INDICES

2.1 Types of Gilts

Over the years the British Government has issued a variety of different types of bonds, generally known as *gilts* (for "gilt-edged securities"). These have included conventional gilts with fixed or variable final redemption dates, gilts with two or four payments per annum, annuities, gilts with sinking funds, floating-rate notes and gilts linked to the retail price index.

The indices and associated statistics contained in the FTSE Actuaries UK Gilts Index Series provide information on both conventional gilts and index-linked gilts.

Whilst the index-linked market is relatively new and all the existing issues work in a fairly consistent way[3], this is not so in the case of conventional gilts. Luckily many of the unusual features are only now found in gilts that are deemed to be too small to be included in the indices. However, the conventional indices still have to allow for "double-dated" and "undated" gilts.

3. The number of decimal places used in the accrued interest and coupon calculations does vary.

2.2 "Double-dated" Gilts

A "double-dated" gilt is one that has been issued with a range of redemption dates, e.g. 7¾% Treasury 2012/15. This means that the British Government has the right to redeem this gilt at any time between 2012 and 2015 provided they give appropriate notice to the holders. The minimum notice period is 3 months.

When including such gilts in the indices, it is assumed that the British Government will always act in its own best interest. In other words, it will redeem the gilt at the earliest date if it feels that it can refinance the debt at a cheaper rate: otherwise, it will wait until the last possible date. Hence, it is conventional to assume that if the redemption yield of the gilt to the earliest date is less than that to the final date, then the gilt will be redeemed at the earliest date. Otherwise, it is assumed that it will be redeemed at the final date. The price of a gilt at which the gross redemption yields to the possible first and last redemption dates are the same is called the "equilibrium" price.

In practice, the Government is unlikely to redeem a gilt early if the price goes marginally over the equilibrium price because, other things being equal, of the costs involved in redeeming the issue and of issuing new debt, and because longer dated debt often commands a higher yield.

It is easy to show that for a normal gilt without a special first or last coupon and which is not being traded ex-coupon that the clean equilibrium price *CEP* is given by:

$$CEP = R \times (1 + C / R)^{1-f} - (1 - f) \times C$$

where:

CEP	=	clean equilibrium price, (i.e. the price without the accrued interest)
R	=	redemption value, (i.e. 100 for conventional gilts)
C	=	normal coupon per cent payable at each future date, which equals the annual coupon rate divided by the coupon frequency (i.e. = 2)
f	=	fraction of a six-monthly period from the settlement date to the next interest date.

This formula gives a clean equilibrium price equal to the redemption amount *R* on any normal coupon date, (i.e. when *f* equals 0 or 1).

Example

For a settlement date of 1 March 2004 consider a gilt which can be redeemed at par (100) any time between 1 March 2008 and 1 March 2011 at the discretion of the issuer. If the quoted clean price of the gilt is above 100, there will be a capital loss at redemption, and so the yield to 1 March 2008 will be less than that to 1 March 2011, with the result that the index calculations will assume that it will be redeemed at the earlier date, i.e. in 4 years time.

Similarly if the clean price of the gilt had been below 100, it would have been treated as if it had an outstanding term of 7 years.

Example

On 1 June 2004 the clean equilibrium price for a gilt, with a 6% annual coupon payable twice a year, which can be redeemed at par any time between 1 March 2008 and 1 March 2011 is given by:

$$f = 92/184 = 1/2$$

$$CEP = 100 \times (1 + 3/100)^{(1-0.5)} - (1 - 0.5) \times 3 = 99.989$$

When looking at indices for specific sectors, such gilts are always put into the relevant sector for their assumed redemption date. This means that if the price changes for such a gilt, from below the equilibrium price to above it or vice versa, then the gilt can move from one sector to another.

In recent years the British Government has not issued any “double-dated” gilts, and it is understood that they do not have any plans to do so again.

2.3 Undated Gilts (Irredeemables)

Undated gilts are gilts where the issuer has not specified a final redemption date. The British Government first issued “undated” gilts a very long time ago: for example, 2½% Consolidated Stock (Consols) had been issued in 1751, as a 3% gilt consolidating a number of other gilts. It was converted to 2½% Consolidated Stock in the early 20th century. It is now repayable only with an Act of Parliament. Undated gilts are also referred to as “irredeemables”, but this is not strictly correct as the government always has the option to call the gilt after a specific date. The last new undated gilt, 2½% Treasury, was issued in October 1946 and the British Government has no plans to issue further quantities of these gilts.

Example

The British Government has had the right to redeem the 3½% War Loan at any time after 1 December 1952 at par, with 3 months notice. There is no final redemption date.

Undated gilts are included in a separate sector of their own. This currently only includes the 3½% War Loan. All other undated gilts appear in the DMO’s rump list and hence are excluded from the FTSE indices.

2.4 **Eligible Conventional Gilts**

All sterling conventional gilts are potentially included in the conventional price indices provided that:

- They are quoted on the Stock Exchange.

- They are of an issue size for there to be an effective market. Gilts in the Debt Management Office's "rump stock" list are regarded as not being sufficiently large for inclusion.

- They do not have a variable interest rate, such as floating rate notes and index-linked gilts.

- They are not still in partly-paid form. Gilts have not been issued in partly-paid form for a number of years. (The last gilt to be issued in partly-paid form was 7% Treasury 2001 A, issued in February 1994.) Such gilts are not included since any change in interest rates has a disproportionate effect on their price. They are included in the indices when they become fully paid, if they are otherwise eligible.

- Some, but not all, conventional gilts are "strippable". That is each gilt may be broken down by a Gilt-edged Market Maker (GEMM) into its separate interest and capital cash flows, and each of these cash flows may then be traded separately. In effect the gilt is broken down into a number of zero coupon gilts.

Example

In July 2003, 8% Treasury 2021, which pays interest semi-annually on 7th June and 7th December, may be stripped into:
36 separate interest payments (payable 7/12/2003, 7/06/2004,……, 7/06/2021), and one capital payment on 7/06/2021

After a gilt has been stripped, it is possible for a GEMM to re-combine the future cash flows again to make an unstripped gilt. This process is referred to as "reconstitution".

The stripped components of gilts are not included in the indices explicitly since the issue sizes of the unstripped original gilts have not been adjusted to allow for the fact that part of the issues have been stripped. This avoids any double counting. In any case, the stripped components are usually too small for there to be a liquid market in them.

Example

In February 2003, 8% Treasury 2015 had the largest amount of any gilt in stripped form, with £255m of nominal stripped, compared with its total amount in issue of £7,377m.

- Gilts issued by the Debt Management Office under their "special repo" arrangements are also not included in the indices.

- Gilts issued with unusual conditions are not included in the indices.

- Convertible and conversion gilt are included in the indices. However, there are currently no such gilts, and the last convertible issue matured in 1997.

In addition to the above restrictions, the following gilts are excluded from the yield index calculations:

- Gilts with less than 1 year to their assumed redemption date. This is because, at least in part, a very small change in the price of such gilts can cause the shape of the constructed yield curve to change considerably.

Example
Exclusion of 6¾% Treasury 2004 from the fitted yield curve

6¾% Treasury 2004 had a redemption date of 26 November 2004. FTSE announced that the gilt was removed from the calculation of the fitted yield curve, but not the price indices, after close of business on Tuesday, 25 November 2003 (i.e. with effect from start of trading on Wednesday, 26 November 2003). It was included in the calculation of the fitted yield curve on 25 November 2003.

- Convertible issues with outstanding conversion options, gilts with substantial sinking funds and gilts with special tax status (as defined by the FTSE Bond Indices Committee) are excluded since their redemption yields are different to gilts of similar outstanding term.

Example

Currently only 5½% Treasury 2008-12, which has a special tax status, is excluded on these grounds from the calculation of the fitted yield curve. However in the past, 3½% Conversion was excluded, until it became a rump stock, because of its sinking fund.

2.5 Eligible Index-linked Gilts

All index-linked gilts are potentially included in the price indices provided that:

- They are quoted on the Stock Exchange.
- They are of an issue size for there to be an effective market. Gilts in the Debt Management Office's "rump stock" list are regarded as not being sufficiently large for inclusion. There are currently no index-linked gilts in the "rump stock" list.
- They are not convertible index-linked issues with outstanding conversion options. There are currently no index-linked convertible gilts – only one has ever been issued and this matured in 1999.
- Any other index-linked gilts would be excluded from the current indices if they were to be issued with conditions which were significantly different from those of the existing index-linked gilts, (e.g. linked to an index other than the RPI).

2.6 Sector Indices

Gilts are grouped together in sectors according to their assumed outstanding terms. The sectors are listed in Rule 1.3.

Gilts will be included in the relevant sectors, provided they are eligible for inclusion in the "All-stock" indices and their assumed redemption date meets the criterion for the sector.

SECTION 2

The outstanding term of a gilt is from the calculation date to the assumed redemption date. Except in the case of "double-dated" and "undated" gilts, there is a fixed redemption date. With double-dated gilts, the assumed redemption date is dependent on whether the price of the gilt is above or below its equilibrium price (see Rule 2.2).

Example

A gilt moves from the 5 to 10 year sector to the under 5 years sector at the end of the day on which it is exactly 5 years from the calculation date to its assumed redemption date, or if this turns out not to be a business day at the beginning of the next business day.

Example

On 7 December 2004 (a Tuesday), 5 ¾% Treasury 2009 will be exactly 5 years from its redemption date of 7 December 2009. The gilt will be included in the 5 -10 year and 5 -15 year sector indices in the calculations for 7 December 2004. Immediately after the initial calculations for that date, it will be removed from the 5 -10 year and 5 -15 year sectors, and will be added to the up to 5 year sector, so that on 8 December 2004 it will appear in the new sector. Thus the change in price between the 7 and 8 December of 5 ¾% Treasury 2009 is reflected in the new sector.

On 7 December 2002 (a Saturday), 7 ¼% Treasury 2007 was exactly 5 years from its redemption date of 7 December 2007. The gilt was included in the 5 -10 year and 5 -15 year sector indices on the 6 December 2002 (a Friday). On the 9 December 2002 (a Monday), it was dropped from the 5 -10 year and 5 -15 year sectors, but added to the up to 5 years sector. No indices are calculated for Saturdays and Sundays.

2.7 Replication

One of the most important factors with any set of indices is a requirement to make it possible for the indices to be replicated. In the case of the FTSE Actuaries UK Gilts Index Series this is achieved by:

- Making available detailed guidelines of how the indices are constructed.
- Producing lists of the constituent gilts together with rules for adding new issues and deleting existing issues.
- Using a public source of reliable prices for their calculation. The prices used in the index calculations are the GEMMA[4] prices compiled and distributed by the UK Debt Management Office each day on behalf of GEMMA. These are available on the DMO wire service pages, as well as on its website: www.dmo.gov.uk.

4. GEMMA is the Gilt-edged Market Makers Association

SECTION 3

3.0 PRICE INDEX

Fundamental to the FTSE Actuaries UK Gilts Index Series and other associated information, is the calculation of the actual gross price indices and how they allow for changes in their constituent gilts.

Changes in the constituents of the sector indices can occur for a variety of reasons. The following rules apply to both conventional and index-linked indices, except where noted.

3.1 Addition of Constituents

Gilts may be added to a sector as a result of:

- A new issue. Such gilts, unless they are partly paid, are added on the business day following the auction, syndication or placement.

Example
Auction of 4 ¾% Treasury 2015

On 16 September 2003 the United Kingdom Debt Management Office announced the auction on a fully paid basis of £2,750 million nominal of 4 ¾% Treasury 2015 on Thursday, 25 September 2003 for settlement on Friday, 26 September 2003. The auction resulted in the acceptance of bids between £100.62 and £100.72. The gilt was added to the FTSE Actuaries UK Gilts Index Series after close of business on 25 September 2003 (i.e. with effect from start of trading on 26 September 2003). The gilt was included in the indices with an initial price of £100.56 (i.e. Thursday's closing market price).

Example
Auction of 2% Index-linked Treasury 2035

On 10 July 2002 the United Kingdom Debt Management Office announced the results of the auction of 2% Index-linked Treasury 2035. It had been allocated at a uniform striking price of £97.80. The gilt was included in the FTSE Actuaries Gilt indices on Thursday, 11 July 2002 at an initial price of £97.01.

Example
Syndicated Offering of 1 ¼ % Index-linked Treasury Gilt 2055

On 22 September 2005 the United Kingdom Debt Management Office announced the results of the syndicated offering of 1 ¼% Index-linked Treasury 2055. It had been priced at £105.29 and the closing market price for 22 September was £105.97. The announcement stated that the gilt would be issued and settled on 23 September 2005. The gilt was included in the FTSE Actuaries UK Gilts Index Series after the close of business on 22 September (i.e. with effect from the start of trading on 23 September 2005) at an initial price of £105.97 (i.e. the closing market price for 22 September).

- Gilts may also be added as a result of conversion from other gilts.

- Gilts may also be added to sectors as a result of the shortening outstanding term of an existing gilt, which was previously in another sector.

> ***Example***
> ***Shortening of 5 ½% Treasury 2008-12***
>
> The earliest date at which 5 ½% Treasury 2008-12 may be redeemed is 10 September 2008. In 2003, this is assumed to be the redemption date as the clean price of the gilt is well over par.
>
> After the close of business on the Wednesday, 10 September 2003 (i.e. with effect from the start of trading on 11 September 2003), the gilt was removed from the 5 – 10 years sector and added to the up to 5 years sector. The gilt left the 5 – 10 years sector with a final clean price of £104.47 and was added to the up to 5 years sector at the same price.

- Similarly "double-dated" gilts may "slide" from one sector to another and back again.

3.2 Removal of Constituents

Gilts may be removed from the indices or a sector as follows:

- Conventional gilts are removed from the indices on their redemption date at the closing price on the previous day. They are removed from the calculation of the fitted yields on the day they reach one year to redemption.

> ***Example***
> ***Deletion 10% Treasury 2003***
>
> 10% Treasury 2003 was redeemed on 8 September 2003. The gilt was removed from the FTSE Actuaries UK Gilts Index Series after close of business on Friday 5 September 2003 (i.e. with effect from the start of trading on 8 September 2003). The closing price on 5 September was £100.00.

- Index-linked gilts are removed from the price indices at their redemption date at the closing price of the previous day.

> ***Example***
> ***Deletion of 2 ½% Index-linked Treasury 2003***
>
> 2½% Index-linked Treasury 2003 was redeemed on 20 May 2003. It was removed from the FTSE Actuaries UK Gilts Index Series after the close of business on Monday, 19 May 2003 (i.e. with effect from start of trading on 20 May 2003) at its closing price of £225.50.

- They are no longer deemed to be of a sufficient size for there to be an efficient market, i.e. they are now in the UK Debt Management Office's "rump stock" list.

- The FTSE Bond Indices Committee considers that they are no longer behaving in the same way as other gilts.

- A gilt has been merged into another gilt. This sometimes occurs when a new tranche of a gilt is issued. Its terms are identical to those of an existing gilt, except that the first interest payment is different. When the new gilt goes ex-dividend the first coupon payment, it becomes identical to the existing gilt. At this stage the two gilts are merged together, i.e. become fungible.

> ***Example***
>
> The last gilt to be re-opened with a tranche in this way was 8% Treasury 2015 A, which was issued in October 1995.

- A gilt has converted wholly into another gilt. It is removed from the indices on the conversion date at the closing price on the previous day.

- The gilt now has a life that is too short for the sector. This is referred to as a "shortener".

> ***Example***
>
> A gilt is removed from the 5 -10 year sector after the close of business on the day when it is exactly 5 years to the assumed redemption date, i.e. if the redemption date is 31 March 2009, it will be included in the 5 -10 year sector closing calculations on 31 March 2004, but will be removed from this sector immediately afterwards.

- The gilt is a "double-dated" gilt which has moved to another sector as a result of a change in its price.

3.3 Alteration to Constituents

The amount in issue of a gilt can change from time to time. This can occur as a result of:

- The government buying back in the market some of the gilt, which it has then cancelled.

- Conversion or a switch auction into another gilt.

- Creation of collateral for cash management operations

> ***Example***
> ***Cancellation of the UK Debt Management Office's Holdings***
>
> On 10 March 2006 the United Kingdom Debt Management Office (DMO) announced the cancellation of its holdings of £182.7 million nominal of 5 ½ % Treasury Stock 2008-2012 and £317.1 million nominal of 7 ½ % Treasury Stock 2012-2015 to take effect from 13 March 2006. The DMO also announced an amendment to the size criterion for "rump" gilts to include gilts with nominal amounts that have been reduced to less than £850 million. Consequently the above gilts were declared "rump" stocks.
>
> The effect on the FTSE Actuaries UK Gilts Index Series was that, after the close of business on Friday, 10 March 2006 (i.e. with effect from the start of trading on Monday 13 March 2006) the above "rump" stocks were deleted from the indices. Please note, the above gilts would have remained in the indices with a reduced nominal if the new outstanding was at least £850 million.

Example
Gilt-edged Conversion

On 5 August 2002 the United Kingdom Debt Management Office announced the result of its conversion offer from 9% Treasury 2008 into 5% Treasury 2008. 87.5% of holders accepted the offer. As a result of the conversion the nominal value of 9% Treasury 2008 decreased from £5,495 million to £686.7 million, and that of 5% Treasury 2008 increased from £3,050 million to £8,971 million. The DMO immediately declared 9% Treasury 2008 to be a "rump" stock, and hence not eligible for the indices.

The effect on the FTSE Actuaries Gilt Index Series was that after the close of business on Monday, 5 August 2002 (i.e. with effect from the start of trading on 6 August 2002) the nominal amount 5% Treasury 2008 was increased to £8,971 million and 9% Treasury 2008 was deleted.

Example
Index-linked Switch Auction

On 19 July 2001 the United Kingdom Debt Management Office announced the results of a switch auction from 2% Index-linked Treasury 2006 into 2 ½% Index-linked Treasury 2016. As a result of this auction, the amount of nominal outstanding of 2% Index-linked Treasury 2006 decreased by £500 million to £2,000 million and that of 2 ½% Index-linked Treasury 2016 increased by £561 million to £5,526 million.

Both gilts remained in the FTSE Actuaries UK Gilts Index Series and their amounts in issue were adjusted after the close of business on Thursday, 19 July 2001 (i.e. with effect from the start of trading on 20 July 2001).

- The issuance of a new tranche of an existing gilt. New tranches are normally fungible from their issue date, although in the past they only became fungible after the new issue went ex-dividend for the first time.

Example
Increase in nominal of 2% Index-linked Treasury 2035

The United Kingdom Debt Management Office announced the issue of a further tranche of £650 million nominal of 2% Index-linked Treasury 2035 by auction on a fully-paid uniform price basis on Tuesday, 23 September 2003 and settlement on Wednesday, 24 September 2003. It was satisfied at a striking price of £100.70.

The amount in issue of the gilt was increased from £1,850 million to £2,500 million after the close of business on 23 September 2003 (i.e. with effect from start of trading on 24 September). The closing price of the gilt on 23 September 2003, at which price the extra tranche was added to the indices, was £101.30.

- The issue of a smaller amount of an existing gilt ("minitaps"), or of very small amounts of all gilts in issue (which is done from time to time for technical reasons).

Example
Creation of collateral for cash management operations

On 6 June 2002 the DMO issued relatively small amounts of a range of gilts for use as collateral in its cash management operations.

The nominal amounts are adjusted at the close of business on the day the UK Debt Management Office issues the gilts.

3.4 Price Index Calculations

In order to ensure that the price index calculations reflect changes to their constituents, they are calculated on a chain-linked basis.

The basic principle used is:

$$Index(today) = Index(yesterday) \times \frac{Market\_Value(today)}{Equivalent\_Value(yesterday)}$$

where: *Market_Value(today)* is the total market value of all the relevant constituents today (i.e. on the calculation date);
Equivalent_Value(yesterday) is the total market value of the same constituents yesterday (i.e. on the previous calculation date) adjusted for changes in capital etc.

The initial value of the index for each original index was 100, but where a sector has been divided into smaller sectors, the new sectors have started with the index value of the larger sector (e.g. the initial index for the 5-10 year and 10-15 year sectors was derived from the 5-15 year sector).

In the above formula, the market value today is the market value, including accrued interest, of all the gilts that were in the sector both yesterday and today. Thus if a new issue is issued "today", it will not be included in the index until "tomorrow" (i.e. the next calculation date), as it is not possible to compare its performance from the previous day.

The basic structure of the index calculations ensures that the constituents of any index and their weights remain constant during the day, but they can change overnight. Throughout this document and in the "Ground Rules", there are references to changes that occur "at or after close of business". These changes do not change today's calculations in any way, but they do change the comparison information in the calculation of tomorrow's indices, (i.e. they change the composition of the denominator in the above formula).

Mathematically, the price index value for sector *s* for day *t*, can be defined as:

$$I_{s,t} = I_{s,t-1} \times \frac{\sum_i N_{i,t} \times P_{i,t}}{\sum_i N_{i,t} \times P_{i,t-1}}$$

where:

$I_{s,t}$	=	today's index value
$I_{s,t-1}$	=	yesterday's index value
$N_{i,t}$	=	adjusted nominal value of gilt *i* today
$P_{i,t}$	=	gross price of gilt *i* today
$P_{i,t-1}$	=	gross price of gilt *i* yesterday

and where the summations are over all gilts in sector *s* today, including those gilts yesterday which have been amalgamated with other gilts today.

The adjusted nominal value today is normally the same as the actual issued nominal today, but there are a few exceptions, e.g. when a new issue comes to the market, it is not included in the indices until the day after issue, as it was not possible to hold the gilt yesterday.

The calculations make use of the gross prices of the gilts, i.e. their quoted clean middle prices plus or minus the calculated accrued interest to the settlement date. The accrued interest calculations are described in Appendix A.

3.4.1 Price Index Calculation Examples

The following examples show how the methodology works and allows for changes in constituents. In all cases a selection of the following gilt data is used, and the prices include accrued interest:

Gilt	Nominal day 1	Price day 1	Nominal day 2	Price day 2	Nominal day 3	Price day 3
A	100	90	100	91	100	92
B	200	95	200	94	200	95
C	-	-	300	99	300	100
D	250	99	0	-	0	-
E	150	85	50	84	50	85
F	200	93	0	-	0	-
G	300	94	500	92	500	94

In the following examples the price index on day 1, $I_{s,1}$ = 120.

Example
Normal case – no change in constituents or amounts in issue

The index for days 1, 2 and 3 consists of just two gilts *A* and *B*, as in the table above.

$$Index(day1) = 120$$

$$Index(day2) = 120 \times \frac{100 \times 91 + 200 \times 94}{100 \times 90 + 200 \times 95} = 119.571$$

$$Index(day3) = 119.571 \times \frac{100 \times 92 + 200 \times 95}{100 \times 91 + 200 \times 94} = 120.857$$

SECTION 3

Example
New gilt issued on day 2

The index on day 1 consisted of two gilts *A* and *B* (see above). On day 2, a new issue *C* is added to the eligible gilts. The calculations are now:

$$Index(day1) = 120$$

$$Index\,(day\,2) = 120 \times \frac{100 \times 91 + 200 \times 94 + 0 \times 99}{100 \times 90 + 200 \times 95 + 0} = 119.571$$

$$Index(day3) = 119.571 \times \frac{100 \times 92 + 200 \times 95 + 300 \times 100}{100 \times 91 + 200 \times 94 + 300 \times 99} = 120.817$$

Note in this case, the calculations on day 2 are identical to those in the previous example, as it was not possible to hold the new gilt on day 1. The new gilt *C* is included in the index calculations on day 2.

Example
Gilt is removed from the index

On day 1, the index consisted of three gilts *A*, *B* and *D*. Gilt *D* is removed from the index on day 2. The calculations are now:

$$Index(day1) = 120$$

$$Index\,(day\,2) = 120 \times \frac{100 \times 91 + 200 \times 94 + 0}{100 \times 90 + 200 \times 95 + 0 \times 99} = 119.571$$

$$Index(day3) = 119.571 \times \frac{100 \times 92 + 200 \times 95}{100 \times 91 + 200 \times 94} = 120.857$$

Gilt *D* does not influence day 2's index calculation, although it did those on day 1. Hence its weight in day 2's calculations must be reduced to 0.

Example
Size of gilt is reduced

On day 1, the index consisted of three gilts *A*, *B* and *E* (see above). On day 2, the size of gilt *E* was reduced to 50, possibly as the result of a conversion or the exercise of a sinking fund. The index calculations are now:

$$Index(day1) = 120$$

$$Index\,(day\,2) = 120 \times \frac{100 \times 91 + 200 \times 94 + 50 \times 84}{100 \times 90 + 200 \times 95 + 50 \times 85} = 119.442$$

$$Index(day3) = 119.442 \times \frac{100 \times 92 + 200 \times 95 + 50 \times 85}{100 \times 91 + 200 \times 94 + 50 \times 84} = 120.744$$

N.B. only the reduced size of gilt *E* has been included in the indices.

SECTION 3

Example
Two existing gilts become fungible

On day 1, the index consisted of four gilts *A*, *B*, *F* and *G* (see above). *F* was a new tranche of gilt *G*, which became fungible with it on day 2, after they both went ex-dividend. On day 2 only the larger gilt *G* is quoted. The index calculations are now:

$$Index(day1) = 120$$

$$Index(day2) = 120 \times \frac{100 \times 91 + 200 \times 94 + 500 \times 92}{100 \times 90 + 200 \times 95 + 200 \times 93 + 300 \times 94} = 118.556$$

$$Index(day3) = 118.556 \times \frac{100 \times 92 + 200 \times 95 + 500 \times 94}{100 \times 91 + 200 \times 94 + 500 \times 92} = 120.642$$

N.B. on day 2, both gilts *F* and *G* are valued at their prices on day 1. They both went ex-dividend by different amounts on day 2, which is allowed for in the XD adjustment calculation.

3.4.2 Price Index Calculations for a "Shortener"

As has been already described, indices are calculated for gilts with specific assumed outstanding term ranges. As a result, over time, it will be necessary to remove gilts from one sector and to put them into a shorter sector. Such gilts are called "shorteners".

The following example describes the effect of a shortener on the calculations of both the longer sector (L) and the shorter sector (S) into which the gilt is placed.

The example uses the following gilt data:

Gilt	Nominal day 1	Price day 1	Nominal day 2	Price day 2	Nominal day 3	Price day 3
A	100	90	100	91	100	92
B	200	95	200	94	200	95
C	300	98	300	99	300	99
D	200	85	200	86	200	87
E	200	96	200	97	200	98

The shorter sector index on day 1, $Index(S, day1)$ is 110, and the longer sector index $Index(L, day1)$ is 120.

SECTION 3

Example
Effect of a "Shortener"

On day 1, *Index* (*L*,*day*1) consists of gilts *A*, *B* and *E*, and *Index*(*S*,*day*1) consists of gilts *C* and *D*. At the end of day 2, gilt *E* moves from the longer index band to the shorter one. The effects of this are as follows:

$$Index(S, day1) = 110$$

$$Index(L, day1) = 120$$

$$Index(S, day2) = 110 \times \frac{300 \times 99 + 200 \times 86}{300 \times 98 + 200 \times 85} = 111.185$$

$$Index(L, day2) = 120 \times \frac{100 \times 91 + 200 \times 94 + 200 \times 97}{100 \times 90 + 200 \times 95 + 200 \times 96} = 120.254$$

$$Index(S, day3) = 111.185 \times \frac{300 \times 99 + 200 \times 87 + 200 \times 98}{300 \times 99 + 200 \times 86 + 200 \times 97} = 111.856$$

$$Index(L, day3) = 120.254 \times \frac{100 \times 92 + 200 \times 95}{100 \times 91 + 200 \times 94} = 121.547$$

<u>On Business Days</u>

A gilt starting in an "over XX years" maturity sector will be moved from that maturity sector to the next-shortest maturity sector after the close of business on the day when its term to maturity is exactly equal to the number of years (XX) in that sector (when it is described as a "timeous shortener"). It will be moved at its closing price on that day. The next-shortest maturity sector may be an "up to XX years" sector or an "XX years to YY years" sector.

Another way of saying this is: If a gilt's *term to maturity* (the exact number of years from the trade date to the gilt's maturity date) falls on a *business day* (when trading is open), and it is currently a constituent in an "over XX years" maturity sector, it will remain in that sector until after the index's value is calculated and disseminated for that day. The following day, when the gilt's term to maturity is now one day shorter, it will become a constituent in the "up to XX years" sector. It may also then become a constituent in an "XX years to YY years" sector, if one exists.

Example
A gilt maturing on 23 August 2011 would be included in the calculation of the value of the "over 5 years" maturity sector on 23 August 2006 (when it had *exactly* 5 years remaining to maturity). After the close of business that day, the gilt would be moved to the "up to 5 years" maturity sector. In other words, it would become a constituent in the shorter maturity sector on the following business day.

A gilt starting in an "XX years to YY years" maturity sector will be moved from that maturity sector to the next-shortest maturity sector after the close of business on the day when its term to maturity is exactly equal to the minimum term (XX years) of that sector (when it is described as a "timeous shortener"). It will be moved at is closing price on that day. The shorter maturity sector may be an "up to XX years" sector or another "XX years to YY years" sector.

SECTION 3

Example
A gilt maturing on 23 August 2023 would be included in the calculation of the value of the "15 -25 years" maturity sector on 23 August 2008 (it had exactly 15 years remaining to maturity). After the close of business on that day, the gilt would be moved to the "up to 15 yeas" maturity sector. In other words, it would become a constituent in the shorter maturity sector on the following business day.

Example
A gilt maturing on 15 October 2013 would remain a constituent in the "over 5 years" maturity sector, the "5 – 10 years" sector, the "5 – 15 years" sector, the "up to 15 years" sector, the "up to 20 years" sector and the All-Stocks sector up until 15 October 2008.

Exactly on 15 October 2008, the gilt would remain a constituent in those maturity sectors and its values would be included in the closing values for those sectors that were calculated and disseminated to clients as of the close of business on 15 October 2008.

On 16 October 2008, when the gilt now has less than 5 years to maturity, it would be included in the "up to 5 years" sector, the "up to 15 years" sector, the "up to 20 years" sector and the All Stocks sector.

On Non-Business Days

If the gilt's term to maturity (the exact number of years from the trade date to the gilt's maturity date) falls on a *non-business day* (Saturday, Sunday or other market holiday), and it is a constituent in an "over XX years" maturity sector or an "XX years to YY years" maturity sector on the business day preceding the non-business day, it will remain in that sector on that preceding business day (since its term to maturity is over XX years as of that business day) until after the index's value is calculated and disseminated for that day. The following *business* day it will become a constituent in the next-shortest maturity sector, since its term to maturity will now will now be shorter (when it is described as a "late shortener").

Example
A gilt maturing on 15 October 2013 would remain a constituent in the "over 5 years" maturity sector, the "5 – 10 years" sector, the "5 – 15 years" sector, the "up to 15 years" sector, the "up to 20 years" sector and the All-Stocks sector up until 15 October 2008.

Exactly on 15 October 2008, the gilt would remain a constituent in those maturity sectors and its values would be included in the closing values for those sectors that were calculated and disseminated to clients as of the close of business on 15 October 2008.

Assuming 16 October was a Saturday, on that day the gilt now has less than 5 years to maturity, it would be included in the "up to 5 years" sector, the "up to 15 years" sector, the "up to 20 years" sector and the All Stocks sector beginning Monday, October 18th.

Assuming 15 October was a Saturday, and the gilt was a constituent in an "over XX years" maturity sector or an "XX years to YY years" maturity sector on the business day preceding that Saturday, it will remain in that sector on that preceding business day (since its term to maturity is over XX years as of that business day) until after the index's value is calculated and disseminated for that day. The following *business* day (i.e. Monday, 17 October) it will become a constituent in the next-shortest maturity sector, since its term to maturity will now be shorter than XX years.

3.4.3 **Price Index Calculations for "Double-Dated" Gilts**

The index calculations for "double-dated" gilts, when their prices are near their equilibrium prices, are more complicated, especially as they can "slide" from one sector to another and back again. As a result, they are removed from, and added to, an index not at their previous prices but at their equilibrium prices. No "double-dated" gilts have been issued by the British Government for some years, and there are now only two such securities (5½% Treasury 2008/12 and 7¾% Treasury 2012/15) still included in the indices. The detailed method of allowing for double-dated gilts ("sliders") is described in the Ground Rules for the Management of the FTSE UK Gilts Index Series.

When a double-dated gilt moves from one sector to another at its equilibrium price, it necessitates the additional calculation of equivalent mean prices for all the other gilts in the sectors. This is shown in the following example.

There are two sectors *L* and *S*. The longer sector *L* on day 1 consists of 3 gilts: *A*, *B* and *E* (which is double-dated), and the shorter sector *S* consists of 2 gilts: *C* and *D*. During day 2 the price of the double-dated gilt rises above its equilibrium price, with the result that it needs to move to the shorter sector.

The gilts in the sectors have the following data:

Gilt	Nominal day 1	Price day 1	Nominal day 2	Price day 2	Nominal day 3	Price day 3
A	100	90	100	91	100	92
B	200	95	200	94	200	95
C	300	98	300	99	300	99
D	200	85	200	86	200	87
E(double-dated)	200	99.5	200	100.5	200	101

The shorter sector's price index on day 1, $Index(S, day1)$ is 110, and the longer sector's index $Index(L, day1)$ is 120.

On day 2, the equilibrium price for the double-dated gilt *E* is 100, hence it needs to move from sector *L* to *S*. In order to do this, mean prices have to be calculated for all the gilts in the sectors using linear interpolation.

As the equilibrium price of the double-dated gilt is midway between its closing prices on days 1 and 2, the mean prices of the other gilts will also be midway between their two prices.

Gilt	Price day 1	Price day 2	Price Mean
A	90	91	90.5
B	95	94	94.5
C	98	99	98.5
D	85	86	85.5
E(double-dated)	99.5	100.5	100.0

The calculations are as follows:

$$Index(S, day1) = 110$$
$$Index(L, day1) = 120$$
$$Index(S, day2) = 110 \times \frac{300 \times 98.5 + 200 \times 85.5}{300 \times 98 + 200 \times 85} \times \frac{300 \times 99 + 200 \times 86 + 200 \times 100.5}{300 \times 98.5 + 200 \times 85.5 + 200 \times 100}$$
$$= 111.173$$
$$Index(L, day2) = 120 \times \frac{100 \times 90.5 + 200 \times 94.5 + 200 \times 100}{100 \times 90 + 200 \times 95 + 200 \times 99.5} \times \frac{100 \times 91 + 200 \times 94}{100 \times 90.5 + 200 \times 94.5}$$
$$= 119.910$$
$$Index(S, day3) = 111.173 \times \frac{300 \times 99 + 200 \times 87 + 200 \times 101}{300 \times 99 + 200 \times 86 + 200 \times 100.5} = 111.671$$
$$Index(L, day3) = 119.910 \times \frac{100 \times 92 + 200 \times 95}{100 \times 91 + 200 \times 94} = 121.199$$

3.4.4 Price Index Methodology Using a Divisor

The Ground Rules for the Management of the FTSE Actuaries UK Gilts Index Series, on the other hand, describe the construction of the price indices in terms of a divisor. With the divisor description, the price index $I_{s,t}$ for sector *s* for today *t* is given by:

$$I_{s,t} = \frac{\sum_i N_{i,t} \times P_{i,t}}{Divisor_{s,t}}$$

where:

	$I_{s,t}$	=	today's index value
	$N_{i,t}$	=	adjusted nominal value of gilt *i* today
	$P_{i,t}$	=	gross price of gilt *i* today
	$Divisor_{s,t}$	=	calculated divisor for sector *s* at time *t*

and where the summation is over all gilts in the sector *s* today, including those gilts yesterday which have been amalgamated with other gilts today.

The divisor in this formula started life as just the market value of the index at the base date divided by 100, (assuming that the index started with a base value of 100). This formulation of the index is easy to understand as long as the constituents of the index do not change in any way, as the index value is just the market value of the constituents now divided by their value at the base date. Unfortunately, whenever there is a change to the constituents, the divisor has to be changed to reflect this. These changes have to allow for constituents being removed from the index (they may have performed differently to that of the index as a whole), the size of constituents changing or new constituents being added. Thus after a while the divisor ceases to have any independent meaning.

The way the divisor changes are calculated is described fully in the Ground Rules.

The two methods of describing the construction of the price index, that is in terms of the change over the previous day and that of using a divisor are equivalent.

SECTION 3

Example

Consider an index I which commenced on date 0. This index has not had any changes to its constituents or changes to capital since its base date.

The index is calculated on dates 0, 1, 2, 3……*n*.

Using the chain-linked method we get:

$$I_0 = 100$$

$$I_1 = 100 \times \frac{\sum_i N_{i,1} \times P_{i,1}}{\sum_i N_{i,0} \times P_{i,0}}$$

$$I_n = 100 \times \frac{\sum_i N_{i,1} \times P_{i,1}}{\sum_i N_{i,0} \times P_{i,0}} \times \frac{\sum_i N_{i,2} \times P_{i,2}}{\sum_i N_{i,1} \times P_{i,1}} \times ....\times \frac{\sum_i N_{i,n} \times P_{i,n}}{\sum_i N_{i,n-1} \times P_{i,n-1}}$$

$$I_n = 100 \times \frac{\sum_i N_{i,n} \times P_{i,n}}{\sum_i N_{i,0} \times P_{i,0}}$$

With the divisor method, as the constituents, and their weightings, have not changed since the base date, the divisor itself has not changed. It is given by:

$$Divisor = \frac{\sum_i N_{i,0} \times P_{i,0}}{100}$$

Hence the index at time n is:

$$I_n = \frac{\sum_i N_{i,n} \times P_{i,n}}{Divisor} = 100 \times \frac{\sum_i N_{i,n} \times P_{i,n}}{\sum_i N_{i,0} \times P_{i,0}}$$

which is the same result as with the chain-linked method.

SECTION 4

4.0 FORMULAE – APPLYING TO BOTH CONVENTIONAL AND INDEX-LINKED GILTS

4.1 Accrued Interest

All gilts in the indices accrue interest at a daily rate for the period which would ensure that the total coupon payment would be accrued by the coupon payment date. This is not visible as all gilts go ex-dividend (XD) 7 business days (or in the case of 3 ½% War Loan 10 business days) before the coupon payment date. If a gilt is sold for settlement during the XD period, the seller retains the coupon.

The accrued interest calculations for securities are described in Appendix A.

The accrued interest for a sector *s* for day *t*, $A_{s,t}$ is calculated as:

$$AI_{s,t} = \frac{\sum_i N_{i,t} \times AI_{i,t}}{\sum_i N_{i,t} \times P_{i,t}} \times I_{s,t}$$

where:

	$I_{s,t}$	=	gross price index of sector *s* at time *t*
	$N_{i,t}$	=	nominal outstanding of gilt *i* at time *t*
	$AI_{i,t}$	=	accrued interest of gilt *i* at time *t*
	$P_{i,t}$	=	gross price of gilt *i* at time *t*

and where the summation is over all gilts in sector *s* at time *t*.

The price index adjustment is to make the calculation comparable with the price index which reflects all the changes in constituents since base date.

Example

Consider a sector which consists of two gilts *A* and *B*, which have the following current information:

Gilt	Nominal	Gross Price	Accrued Interest
A	100	95	2
B	200	90	3

The current price index is 150.

The accrued interest for the sector is then:

$$Accrued\_Interest(Sector) = \frac{100 \times 2 + 200 \times 3}{100 \times 95 + 200 \times 90} \times 150 = 4.364$$

4.2 Gross or "Dirty" Prices

Gilts are quoted in the market with clean prices, i.e. at a price which excludes any accrued interest. This means that the price quotation does not have to be modified every day to allow for the daily effect of accrued interest. When a gilt is traded, the transaction is executed at a gross or "dirty" price, i.e. the clean price of the gilt plus/minus any accrued interest to the appropriate settlement date.

The settlement date for a gilt is usually the next business day. Thus normally on a Thursday, the accrued interest is calculated to the following day, the Friday, but on a Friday it is calculated to the following Monday (normally the next business day). In this case, a transaction executed on the Friday would include 3 days more accrued interest than one executed on the previous Thursday (instead of just 1 day more), assuming the gilt had not gone ex-dividend.

Gross prices are used in all the index calculations.

4.3 XD Adjustment

The effect of a gilt in an index going ex-dividend (XD) is to reduce the price index of the sector, since if the market quotation of the gilt is unchanged, its gross price will have decreased. Sometimes, as in the case of the Total Return Index, it is desirable to measure the effect of re-investing these coupons. This is achieved by calculating the XD adjustment factor.

The XD adjustment for sector *s* today, $XD_{s,t}$ is calculated as:

$$X_{s,t} = \frac{\sum_i N_{i,t-1} \times XD_{i,t}}{\sum N_{i,t-1} \times P_{i,t-1}} \times I_{s,t-1}$$

where:

$I_{s,t-1}$	=	gross price index of sector *s* at time *t*-1
$N_{i,t-1}$	=	nominal outstanding of gilt *i* at time *t*-1
$XD_{i,t}$	=	quantum interest for gilt *i* that has gone ex-dividend between the last day on which the indices were calculated, and the opening of today *t*
$P_{i,t-1}$	=	gross price of gilt *i* at time *t*

and where the summation is over all gilts in sector *s* at time *t*-1.

Note that the previous day's values are now used to weight the calculations, as for the ex-dividend to have an effect the gilts must have been held on the previous day.

$XD_{i,t}$ for gilt *i* is 0 if the gilt has not gone ex-dividend between day *t*-1 and day *t*, otherwise it is the percentage coupon payment that has gone XD.

Example

Consider a 5% gilt that goes XD on 4th April of a standard semi-annual coupon payment of 2.5%.

On 3rd April, the XD adjustment XD_i is 0
On 4th April, the XD adjustment XD_i is 2.5
On 5th April, the XD adjustment *XDi* is 0.

Example

Consider a sector which consists of two gilts A and B. On the previous calculation date A and B had respectively 100 and 200 of nominal in issue and gross prices of 95 and 90. Immediately prior to today gilt A goes XD with a payment of 2.5%, and the price index of the sector on the previous day is 140. The calculation of the XD adjustment for the sector today is:

$$XD\_adjustment(Sector) = \frac{100 \times 2.5 + 200 \times 0}{100 \times 95 + 200 \times 90} \times 140 = 1.273$$

The published figure for sector *s* for day *t* is the XD adjustment for the year to date. This figure is just the sum of all the daily figures for the current year up to date *t*.

4.4 Total Return Index

The Total Return Index for sector *s* for day *t*, $R_{s,t}$, is calculated from the sector Price Index and XD adjustment by:

$$R_{s,t} = R_{s,t-1} \times \frac{I_{s,t}}{I_{s,t-1} - XD_{s,t}}$$

where:

where:	$I_{s,t}$	=	sector *s* price index for day *t*
	$XD_{s,t}$	=	XD adjustment factor for sector *s* for day *t*.

As the XD adjustment factor for a sector is zero whenever there are no coupon payments on the day in the sector. Hence the percentage movement in the total return index is identical to that of the gross price index whenever there are no coupon payments in the sector.

Example

If 3 weeks ago the price index of a sector was 110 and its total return index was 140, and its price index is now 120, then provided there have been no gilts in the sector which have gone ex dividend in the intervening period, the current total return index is:

140 x 120 / 110 = 152.727

4.5 Number of Securities

The number of securities is just the number of different securities today in the index sector.

4.6 Sector Weight

The weight of a sector *s* is just its market value today $M_{s,t}$ divided by the market value of All-stocks sector $M_{A,t}$. That is:

$$M_{s,t} = \sum_{i \in s} N_{i,t} \times P_{i,t}$$

$$M_{A,t} = \sum_{i \in A} N_{i,t} \times P_{i,t}$$

where: $N_{i,t}$ = nominal outstanding of gilt *i* at time *t*
$P_{i,t}$ = gross price of gilt *i* at time *t*

and where the summations are over all gilts in sector *s* and the combined sector *A* respectively at time *t*.

Then:

$$W_{s,t} = 100 \times M_{s,t} / M_{A,t}$$

where $W_{s,t}$ is the percentage weight of sector s today.

Example

Assuming the All-stocks sector consists of two sectors *X* and *Y*. Both sectors consist of two gilts *A* and *B*, and *C* and *D* respectively. At the calculation date the gilts have the following data:

Gilt	Nominal Outstanding	Clean Price	Accrued Interest
A	100	90	2
B	300	95	1
C	200	80	0
D	400	85	4

The market values M_X, M_Y of sectors *X* and *Y* are:

$$M_X = 100 \times (90 + 2) + 300 \times (95 + 1) = 38000$$
$$M_Y = 200 \times (80 + 0) + 400 \times (85 + 4) = 51600$$

Hence:

$$W_X = \frac{38000}{38000 + 51600} \times 100 = 42.41\%$$
$$W_Y = \frac{51600}{38000 + 51600} \times 100 = 57.59\%$$

where W_X and W_Y are the weights of the two sectors.

SECTION 5

5.0 FORMULAE – APPLYING TO CONVENTIONAL GILTS ONLY

5.1 Gross Redemption Yield

As all the gilts included in both the conventional and the index-linked indices pay coupons twice a year, it is normal in the market place to quote redemption yields which are compounded on a six-monthly basis. This convention is adopted here.

The redemption yield of a conventional gilt with 2 semi-annual coupon payments, compounded annually can be determined from the following formula:

$$P = \sum_{t=1}^{T} \frac{CF_t}{(1 + y/2)^t}$$

$$= \sum_{t=1}^{T} CF_t \times v^t$$

where:

	P	=	gross price (i.e. clean price plus/minus accrued interest)
	T	=	number of future cash flows, i.e. future coupon payments and the final redemption amount
	CF_t	=	t^{th} cash flow, i.e. interest and capital payments
	t	=	time in periods (i.e. six-monthly coupon periods) to the t^{th} cash flow
	v	=	discounting factor, i.e. $v = 1/(1 + y/2)$
	y	=	required redemption yield.

This formula is used for gilts with more than one coupon payments remaining. The redemption yield for index-linked gilts is calculated in a similar way but with the following differences:

- In determining the standard equation, future cash flows may be grossed up by the assumed inflation rate;
- The resulting calculated discount factor has to be netted down by the assumed rate of inflation.

The discounting period t used for each cash flow CF_t is the number of interest periods and fraction of periods to the relevant payment date.

In the above equation, all the cash flows CF_t, other than the first one in the case of new issues and the last one, are normally equal to half the annual coupon of the security, as the gilts pay interest twice a year. The last cash flow is normally equal to the redemption amount plus half the annual coupon.

Example

If it is 7 August 2011 and a cash flow payment of 8 is due on 7 September 2012, then it is discounted for two and a bit periods in the calculations – a fractional period from 7 August 2011 to 7 September 2011, a whole period from 7 September 2011 to 7 March 2012 and another whole period from 7 March 2012 to 7 September 2012. The fractional period has a value 31/184 as there are 31 and 184 days from 7 August 2011 to 7 September 2011 and 7 March 2011 to 7 September 2011 respectively.

This cash flow will thus have a contribution of 8 x $v^{2.168}$ to the right hand side of yield equation, (31/184 = 0.168).

> ***Example***
>
> Consider an 8% gilt which pays coupons semi-annually, which is priced at 104.284 on its coupon date 18 months before its redemption.
>
> A holder of this gilt until redemption will receive the following interest and capital payments:
>
> 4 in 6 months time (1 period t)
> 4 in 12 months time
> 104 in 18 months time (coupon plus the capital repayment).
>
> To calculate the redemption yield y for this gilt, it is necessary to solve the equation:
>
> $$104.284 = 4 \times v + 4 \times v^2 + 104 \times v^3$$
>
> where: $y = 2 \times (1/v - 1)$
>
> This gives $v = 0.9756$ and $y = 0.05 = 5\%$

The gross redemption yield for a sector is calculated by discounting all the cash flows from the gilts in the sector at the same rate, i.e. it performs the redemption yield calculation as if the sector were just a single gilt. No distinction is made between cash flows emanating from different gilts.

> ***Example***
>
> Consider a sector which consists of just two gilts. Gilt A has a 6% coupon and pays interest on 7 March and 7 September each year until it is redeemed on 7 September 2009. Gilt B has a 4% coupon and pays interest on 7 June and 7 December until it is redeemed on 7 December 2006. On 7 June 2003 gilt A has outstanding £200m of nominal value and a gross price, including accrued interest, of 105, whereas gilt B £100m outstanding and a gross price of 95.
>
> Extending the formula given above, the following has to be solved to calculate the sector redemption yield:
>
> $200 \times 105 + 100 \times 95 = 200 \times v^{0.5} \times (3 + 3 \times v + 3 \times v^2 + \text{---} + 3 \times v^{13} + 100 \times v^{13}) + 100 \times (2 \times v + 2 \times v^2 + \text{---} + 2 \times v^7 + 100 \times v^7)$
>
> where v is the discount factor, i.e. $v = 1/(1 + y/2)$, and y is the required sector redemption yield.

However, when reporting yield, duration and convexity figures for individual gilts, FTSE follows market convention and uses different formulae for bonds in their final coupon period. A gilt enters the final coupon payment period only after the interest date has passed, at which point the yield, duration and convexity calculations switch to using simple interest.

The definition of simple interest during the last coupon payment of a gilt is as follows:

$$SimpleInterest_{annualised} = \frac{\left(\dfrac{FinalPayment}{Dirtyprice} - 1\right)}{YearstoMaturity}$$

Equivalently, the price of a bond as a function of simple interest during the last payment period is given by:

$$P = \frac{CF_T}{(1 + r \times f)}$$

where: P = gross price (i.e. clean price plus/minus accrued interest)
r = annualised simple interest rate
CF_T = final cash flow (coupon and principal)
f = fraction of a year to maturity

5.2 Macaulay Duration

The Macaulay duration of a bond and hence by extension that of a sector, is just the average life, expressed in years, of all the future cash flows after they have been discounted by the gross redemption yield. It thus measures how long, on average, the bond will be outstanding. The Macaulay duration is sometimes just referred to as the duration.

The duration D_{Mac} of a bond is derived by solving an equation of the form:

$$D_{Mac} = \frac{1}{P}\sum_{t=1}^{T}\frac{t \times CF_t}{(1 + y/2)^t} \;/\; frequency$$

$$= \frac{\sum_{t=1}^{T} CF_t \times v^t \times t}{\sum_{t=1}^{T} CF_t \times v^t} \;/\; frequency$$

where: D_{Mac} = Macaulay duration in years
T = number of future cash flows, i.e. future coupon payments and the final redemption amount
CF_t = t^{th} cash flow, i.e. interest and capital payments
t = time in periods (i.e. six-monthly coupon periods) to the t^{th} cash flow
frequency = number of payments per year, i.e. 2 for gilts. This is required as the duration is normally expressed in years.
v = discounting factor, i.e. $v = 1/(1 + y/2)$
y = redemption yield of the security or sector.

Example

Consider a sector that has a gross redemption yield y, and an associated discounting factor v, where $v = 1 / (1 + y / 2)$. It has anticipated future cash flows of 100, 200 and 300 in 1, 2 and 3 six-monthly periods respectively, then the duration D_{Mac} of the sector is:

$$D_{Mac} = \frac{100 \times v + 200 \times v^2 + 300 \times v^3}{100 + 200 + 300} / 2 \, years$$

Example

Using, as an example, the same gilt as that used in the redemption yield calculation, i.e. an 8% gilt yielding 5% with exactly 18 months to go to redemption. As y =0.05, and v = 0.9756, the duration D_{Mac} is given by:

$$D_{Mac} = \frac{4\times 0.9756\times 1 + 4\times 0.9756^2\times 2 + 104\times 0.9756^3\times 3}{4\times 0.9756 + 4\times 0.9756^2\times 104\times 0.9756^3} / 2 = 1.44 years$$

When calculating the Macaulay duration of an individual bond in the final payment period, the following formula is used:

$$D_{Mac} = f$$

where f is the fraction of a year to maturity.

5.3 Modified Duration

The modified duration of a bond or index measures how sensitive the yield is to a change in price and vice versa. It is defined as the percentage change in price for a unit change in yield. Hence, modified duration is a measure of volatility.

The modified duration D_{Mod} for a bond is given by:

$$D_{Mod} = -\frac{dP}{dy}\times\frac{1}{P}$$

where:

P	=	gross price
dP	=	small change in price
dy	=	corresponding small change in yield.

For a gilt paying semi-annual coupons, modified duration is:

$$D_{Mod} = \frac{D}{(1 + y/2)}$$
$$= D\times v$$

where:

D_{Mac}	=	Macaulay duration of the bond or sector
v	=	discounting factor, i.e. $v = 1 / (1 + y / 2)$
y	=	gross redemption yield of the bond or sector.

Whenever an index has a positive yield, the modified duration will be less than the Macaulay duration.

Example

If an index has a gross redemption yield of 4.5% and a Macaulay duration of 3 years, then it will have a modified duration of:

$$3\times v = 3/(1 + 0.045/2) = 2.934$$

> ***Example***
>
> If a gilt yielding 5% p.a. compounded semi-annually has a Macaulay duration of 1.44, then its modified duration D_{Mod} is given by:
>
> $$D_{Mod} = 1.44/(1+0.05/2) = 1.405$$

The modified duration D_{Mod} for a bond in the final period is:

$$D_{Mod} = \frac{f}{(1 + r \times f)}$$

where: r = annualised single interest rate
f = fraction of a year to maturity.

5.4 Convexity

The relationship between price and redemption yield is not linear, but curved. In a similar way to modified duration, which measures the slope of this relationship, convexity measures the degree of curvature: it is related to the second derivative d^2P/dy^2. The formula below is for Macaulay Convexity with 2 semi-annual coupon payments:

$$C_{Mac} = \frac{1}{P}\sum_{t=1}^{t=T}\frac{t^2 \times CF_t}{(1+y/2)^t} \Big/ frequency^2$$

$$= \frac{\sum_{t=1}^{T} CF_t \times v^t \times t^2}{\sum_{t=1}^{T} CF_t \times v^t} \Big/ frequency^2$$

where:

C_{Mac}	=	Macaulay Convexity
T	=	number of future cash flows, i.e. future coupon payments and the final redemption amount
CF_t	=	t^{th} cash flow, i.e. interest and capital payments
t	=	time in periods (i.e. six-monthly coupon periods) to the t^{th} cash flow
frequency	=	number of payments per year, i.e. 2. This is required if the convexity is to be expressed in years^2.
v	=	discounting factor, i.e. $v = 1/(1 + y/2)$
y	=	redemption yield of the security or sector.

> ***Example***
>
> Using, as an example, the same gilt as that used in the redemption yield calculation, i.e. an 8% gilt yielding 5% with exactly 18 months to go to redemption.
>
> As y =0.05, and v = 0.9756, the convexity C_{Mac} is given by:
>
> $$C_{Mac} = \frac{4\times0.9756\times1 + 4\times0.9756^2\times2^2 + 104\times0.9756^3\times3^2}{4\times0.9756 + 4\times0.9756^2 \times 104\times0.9756^3} / 4 = 2.130$$

The formula for Macaulay Convexity for a bond with a single remaining coupon payment is:

$$C_{Mac} = f^2$$

Where *f* is the fraction of a year to maturity or the Macaulay Duration of the gilt

5.5 Fitted Yields for Conventional Gilts

Fitted yields give the theoretical yields for conventional gilts with outstanding terms of exactly 5, 10, 15, 20 and 25 years. A fitted yield is also calculated for undated securities.

This is achieved by calculating for all the acceptable dated conventional gilts, including the undated (irredeemable) issues but excluding all gilts with less than a year to maturity, the best fit curve of their gross redemption yields against their outstanding terms. The form of the yield curve, which allows for two changes of direction, is:

$$y(m) = A + B \times e^{-Cm} + D \times e^{-Em}$$

where *A, B, C, D* and *E* are parameters. The curve is found by minimising:

$$S(A, B, C, D, E) = \sum_i N_i \times P_i \times (y(m_i) - y_i)^2$$

where:			
	y	=	gross redemption yield of the i^{th} gilt
	m	=	outstanding term to redemption of the i^{th} gilt
	$y(m)$	=	yield given by the curve for outstanding term m_i
	N_i	=	amount in issue of the i^{th} gilt
	P_i	=	gross price of the i^{th} gilt

and where the summation is over all the acceptable dated gilts. All the acceptable dated gilts in the price indices which have an outstanding term of at least one year are included in the calculation of the fitted yields.

Fitted yields are published for terms of 5, 10, 15, and 20 years.

For undated gilts (irredeemables) the yield $y(\infty)$, is also published and calculated as:

$$y(\infty) = \frac{\sum_i N_i \times P_i \times y_i}{\sum_i N_i \times P_i}$$

That is, it is just the average yield of the undated gilts weighted by their outstanding market values.

SECTION 6

6.0 FORMULAE – APPLYING TO INDEX-LINKED GILTS ONLY

6.1 Coupon and Redemption Amount Calculations

Both coupon and redemption payments for all index-linked gilts currently in issue are linked to the UK retail prices index (RPI). Indexation for gilts issued prior to 22 September 2005 is based on an 8-month lag. New gilts issued on or after that date are based on a 3-month lag. Examples in the next section are based on 8-month lag. Examples for gilts with 3-month lag can be found in Section 7.

Example

Consider an index-linked gilt which is issued on 26 January 2004 with a 4% coupon, payable semi-annually, and which will be redeemed 10 years after issue on 26 January 2014 at par indexed by the RPI. If the RPI for May 2003 (i.e. 8 months prior to the issue date of January 2004) is 200, and the RPI values are as in the table below, the payments will be as shown.

	Payment Date	RPI Date	RPI Value	Payment
Interest	26-07-04	Nov-03	200.5	2 x 200.5/200 = 2.005
Interest	26-01-05	May-04	201.0	2 x 201.0/200 = 2.010
Interest	26-07-05	Nov-04	202.0	2 x 202.0/200 = 2.020
Interest	26-01-14	May-13	280.0	2 x 280.0/200 = 2.800
Capital	26-01-14	May-13	280.0	100 x 280/200 = 140

As has been mentioned before, the calculations are based on assumptions that in future the UK Retail Prices Index will rise by 0%, 3%, 5% and 10% per annum. However, if we are looking at a cash flow which is to be paid in one year's time, and we are assuming a future inflation rate of 10% p.a., this does not mean that this value will be 10% more than its value today because of the index lagging.

Example

In August 2003, for illustrative purposes assume that the recent RPI values have been:

Nov 2002	199.0
Dec 2002	199.5
Jan 2003	200.0
Feb 2003	201.0
Mar 2003	201.1
Apr 2003	201.5
May 2003	203.0
Jun 2003	204.0
Jul 2003	204.5
Aug 2003	205.0

Any payment due in September 2003, irrespective of whether it is an interest or capital payment, will be based on the January 2003 RPI, those due in October 2003 on the February 2003 RPI etc. Irrespective of what assumptions are assumed about future inflation rates, these payments are unchanged.

If an annual inflation rate of 10% is assumed, then the inflation for each future month is assumed to increase by the ratio *r*, where *r* is given by:

$$r = (1 + 10/100)^{1/12} = 1.00797$$

Example

Using the above example, with 10% inflation the assumed future RPI figures are:

Sep 2003	205 x 1.00797 = 206.6
Oct 2003	205 x 1.00797^2 = 208.3
Nov 2003	205 x 1.00797^3 = 209.9
Dec 2003	205 x 1.00797^4 = 211.6
Jan 2004	205 x 1.00797^5 = 213.3 etc

Example

Again using the above example RPI, if a gilt which is issued with a 4% coupon, based on a base RPI of 198.0, pays standard six monthly interest payments in January and July, then the payments will be:

Jul 2003	2 x Nov 02 RPI / 198 = 2 x 199 / 198 = 2.010%
Jan 2004	2 x May 03 RPI / 198 = 2 x 203 / 198 = 2.050%
July 2004	2 x Est. Nov 03 RPI / 198 = 2 x 205 / 198 = 2.070% assuming no future inflation
July 2004	2 x Est. Nov 03 RPI / 198 = 2 x 209.9 / 198 = 2.120% assuming future inflation of 10%

6.2 Real Redemption Yield Calculations

Real redemption yields for index-linked gilts are calculated in a similar way to those for conventional gilts, although there are two differences:

- In solving the standard equation, future cash flows may be grossed up by the assumed inflation rate as described in the previous section;
- The resulting calculated discount factor has to be netted down by the assumed rate of inflation.

The real redemption yield for index-linked gilts is derived by solving an equation of the form:

$$P = \sum_{i=1}^{n} CF_i \times v^{Li}$$

where: P = gross price (i.e. clean price plus/minus accrued interest)
n = number of future cash flows, i.e. future coupon payments and the final redemption amount
CF = ith cash flow, i.e. interest and capital payments, grossed up by the assumed future inflation rate as described in the previous section
L_i = time in periods (i.e. six-monthly coupon periods) to the ith cash flow
V = discounting factor with the future cash flows grossed up by the assumed inflation rate if necessary, i.e. $v = 1/((1 + ry/2) \times r^6)$
ry = required real redemption yield
r = monthly inflation ratio, i.e. $r = (1 + i)^{1/12}$
I = assumed annual inflation rate, e.g. $i = 0.05$ for 5% inflation

Solving this equation gives a real redemption yield which has been compounded semi-annually.

In the above equation, the first cash flow amount CF_1 is known as it is based on an already known RPI.

Example

On 19 January 2004, consider an index-linked gilt with a coupon of 4%, payable semi-annually, based on a base RPI of 180, which will be redeemed in 18 months' time on 19 July 2005. The gilt will make interest payments on 19 July 2004, 19 January 2005 and 19 July 2005. The next interest payment will be based on the known RPI of 240.0 for November 2003 (i.e. 8 months before July 2004).

The latest published monthly RPI of 241.0 is for December 2003.

The scheduled cash flows for the gilt based on assumed future inflation rates of 0%, 5% and 10% are:

Assumed inflation rate	**0%**	**5%**	**10%**
Interest in 6 months	C	C	C
Interest in 12 months	A	$A \times r^5$	$A \times s^5$
Interest in 18 months	A	$A \times r^{11}$	$A \times s^{11}$
Capital in 18 months	B	$B \times r^{11}$	$B \times s^{11}$

where: C = 2 x 240 / 180 = 2.667%
A = 2 x 241 / 180 = 2.678%
B = 100 x 241 / 180 = 133.889%
r = $(1 + 0.05)^{1/12}$
s = $(1 + 0.10)^{1/12}$

The discount rates v_0, v_5 and v_{10} for assumed inflation rates of 0%, 5% and 10% respectively are calculated by solving the following equations:

$$P = C \times v_0 + A \times v_0^2 + (A + B) \times v_0^3$$
$$P = C \times v_5 + A \times r^5 \times v_5^2 + (A + B) \times r^{11} \times v_5^3$$
$$P = C \times v_{10} + A \times s^5 \times v_{10}^2 + (A + B) \times s^{11} \times v_{10}^3$$

where: $v_j = 1/((1 + ry_j/2) \times r^6)$
and ry_j is the real redemption yield at the relevant assumed inflation rate.

SECTION 6

Real redemption yields for index-linked sectors are calculated, in a similar way to that for conventional sectors, from the projected combined future cash flows of the gilts in the sector in the same way as for a single security.

6.3 Other Calculations

Macaulay Duration, Modified Duration and Convexity for Index-linked gilts are calculated in the same way as for conventional gilts, but these calculations now use the discounting factor $v = 1/((1 + ry/2) \times r^{6})$ from the calculation of the real redemption yield.

It is important to realize that the discounting factor v is NOT $1/(1 + ry/2)$ where ry is the real yield.

The relationship between Macaulay's duration MD and modified duration D is still:

$$MD = D \times v$$

SECTION 7

7.0 CALCULATIONS FOR INDEX-LINKED GILTS WITH A 3-MONTH LAG

7.1 Indexation Methodology

The indexation methodology for the Index-linked gilts with a 3-month lag is as follows:

Index Ratio $_{Date}$ = $\dfrac{\text{Ref } RPI_{Date}}{\text{Ref } RPI_{First\ Issue\ Date}}$ (Rounded to 5 decimal places)

Where

Index Ratio $_{Date}$ refers to the Index Ratio for a given date

Ref RPI $_{Date}$ = Ref RPI$_{M}$ + $\left(\dfrac{t-1}{D}\right)$ [Ref RPI$_{M+1}$ - Ref RPI$_{M}$]

Where:

D = Number of days in calendar month in which the given date falls.

t = Calendar days corresponding to the given date.

RPI$_{M}$ = Reference RPI for the first day of the calendar month in which the given date falls.

Ref RPI$_{M+1}$ = Reference RPI for the first day of the calendar month immediately following the given date.

Ref RPI$_{First\ issue\ Date}$ = = Ref RPI$_{M}$ + $\left(\dfrac{t-1}{D}\right)$ [Ref RPI$_{M+1}$ - Ref RPI$_{M}$]

This is similar to the Ref RPI Date formula except 'M' denotes the calendar month of the issue date.

The worked examples below are based on a hypothetical 4% coupon gilt issued on the 26 January 2004 with a maturity date of 26 January 2014. Interest is payable semi-annually in January and July.

7.2 Calculating Coupon Payments

Semi-annual Coupon Payment = $\dfrac{C}{2}$ x Index Ratio$_{Div\ payment\ date}$ (Rounded to 6 decimal places)

Where:

C = Annual Coupon Amount

Index Ratio$_{Div\ payment\ date}$ = $\dfrac{\text{Ref } RPI_{DividendPaymentDate}}{\text{Ref } RPI_{First\ Issue\ Date}}$

7.2.1 Calculating the Reference RPI for the Coupon Payment Date

$$\text{Ref RPI}_{\text{Div payment date}} = \text{Ref RPI}_M + \left(\frac{t-1}{D}\right)[\text{Ref RPI}_{M+1} - \text{Ref RPI}_M]$$

Where:

D = Number of days in calendar month in which the coupon payment date falls.

t = Calendar days corresponding to the given date.

RPI_M = Reference RPI for the first day of the calendar month in which the given date falls.

Ref RPI_{M+1} = Reference RPI for the first day of the calendar month immediately following the given date.

For example, assuming the RPI values for April 2004 and May 2004 are 203.0 and 203.5 respectively, the reference RPI for 26 July 2004 is calculated as follows:

Ref $RPI_{26\ July\ 2004}$ = Ref $RPI_{1\ July\ 2004}$ + (26-1)/31 [Ref $RPI_{1\ August\ 2004}$ – Ref $RPI_{1\ July\ 2004}$]

= RPI $April_{2004}$ + (25/31) [$RPI_{May\ 2004}$ – $RPI_{April\ 2004}$]

= 203.0 + (25/31)[203.5 – 203.0]

= 203.40323 (Rounded to 5 decimal places)

7.2.2 Calculating the Reference RPI for the First Issue Date

$$\text{Ref RPI}_{\text{First issue Date}} = = \text{Ref RPI}_M + \left(\frac{t-1}{D}\right)[\text{Ref RPI}_{M+1} - \text{Ref RPI}_M]$$

Where:

D = Number of days in calendar month in which the issue date falls.

t = Calendar days corresponding to the given date.

RPI_M = Reference RPI for the first day of the calendar month in which the issue date falls.

Ref RPI_{M+1} = Reference RPI for the first day of the calendar month immediately following the issue date.

Assuming the RPI value of 202.0 for October 2003 and 202.5 for November 2003, the reference RPI for 26 January 2004 (i.e. first issue date) is calculated as follows:

Ref $RPI_{26\ January\ 2004}$ = Ref $RPI_{1\ January\ 2004}$ + (26-1)/31 [Ref $RPI_{1\ February\ 2004}$ – Ref $RPI_{1\ January\ 2004}$]

= $RPI_{October\ 2003}$ + (25/31) + [RPI $_{November\ 2003}$ – Ref $RPI_{October2003}$]

= 202.0 + (25/31) + [202.5 – 202.0]

= 202.40323 (Rounded to 5 decimal places)

7.2.3 **Calculating the Index Ratio Coupon Payment Date**

Recall The Index Ratio div payment date = Ref RPI div payment date/Ref RPI First Issue date

=203.40323/202.40323
=1.00494 (Rounded to 5 decimal places)

7.2.4 **Calculating the Coupon Payment Amount**

Recall;

Semi-annual Coupon Payment = Annual coupon/2 * Index Ratio $_{\text{Div payment date}}$
= 4/2 x1.00494
= £2.009880 (Rounded to 6 decimal places)

7.3 **Calculating the Redemption Payment**

Redemption Payment = 100 x $\text{Ratio}_{\text{Redemption date}}$

Index $\text{Ratio}_{\text{Redemption date}}$ = Ref $\text{RPI}_{\text{Redemtion date}}$/Ref $\text{RPI}_{\text{First issue date}}$

$$\text{Ref RPI}_{\text{Redemtion date}} = \text{Ref RPI}_{M} + \left(\frac{t-1}{D}\right)[\text{Ref RPI}_{M+1} - \text{Ref RPI}_{M}]$$

Where:

D = Number of days in calendar month in which the redemption date falls.

t = Calendar days corresponding to the given date.

RPI_M = Reference RPI for the first day of the calendar month in which the redemption date falls.

Ref RPI_{M+1} = Reference RPI for the first day of the calendar month immediately following the redemption date.

Assuming the RPI value of 280.0 for October 2013 and 280.5 for November 2013, the reference RPI for 26th January 2014 (i.e. redemption date) is calculated as follows:

Ref $\text{RPI}_{26\text{ January }2014}$ = Ref $\text{RPI}_{1\text{ January }2014}$ + (26-1)/31 [Ref $\text{RPI}_{1\text{ February }2014}$ – Ref $\text{RPI}_{1\text{ January }2014}$]

= $\text{RPI}_{\text{October }2013}$ + (25/31) x [RPI $_{\text{November }2013}$ – $\text{RPI}_{\text{October }2013}$]

= 280.0 + (25/31) x [280.5 – 280.0]

= 280.40323 (Rounded to 5 decimal places)

The Ref $\text{RPI}_{\text{First issue date}}$ calculated as in previous examples giving a value of 202.40323

Index $\text{Ratio}_{\text{Redemption date}}$ = 280.40323/202.40323
= 1.38537 (Rounded to 5 decimal places)

Redemption Payment = 100 x 1.38537
=£138.537000 (Rounded to 6 decimal places)

7.4 **Cashflows based on various Inflation Assumptions**

As has been mentioned before, yield, duration, modified duration and convexity calculations are based on assumptions that in future the UK Retail Prices Index will rise by 0%, 3%, 5% and 10% per annum. However, if we are looking at a cash flow which is to be paid in one year's time, and we are assuming a future inflation rate of 10% p.a., this does not mean that this value will be 10% more than its value today because of the index lagging.

Example

In August 2004, for illustrative purposes assume that the recent RPI values have been:

Oct 2003	202.0
Nov 2003	202.5
Dec 2003	200.0
Jan 2004	201.0
Feb 2004	201.1
Mar 2004	201.5
Apr 2004	203.0
May 2004	203.5
Jun 2004	204.5
July 2004	204.3
Aug 2004	205.0

Any payment due in January 2005, irrespective of whether it is an interest or capital payment, will be based on the RPI values for October 2004 and November 2004 since the Ref RPI for January is based on the RPIs for those months. Irrespective of what assumptions are assumed about future inflation rates, these payments are unchanged.

If an annual inflation rate of 10% is assumed, then the inflation for each future month is assumed to increase by the ratio r, where r is given by:

$$r = (1 + 10/100)^{1/12} = 1.00797$$

Example

Using the above example, with 10% inflation the assumed future RPI figures are:

Sep 2004	$205 \times 1.00797 = 206.6$
Oct 2004	$205 \times 1.00797^2 = 208.3$
Nov 2004	$205 \times 1.00797^3 = 209.9$

etc.

Example

Again using the above example RPI, if a gilt which is issued with a 4% real coupon, then the payment on 26 January 2005 be:

Semi-annual Coupon Payment = $\frac{C}{2}$ x Index Ratio$_{\text{Div payment date}}$ (Rounded to 6 decimal places)

The Index Ratio$_{\text{div payment date}}$ = Ref RPI$_{\text{Div payment date}}$/Ref RPI$_{\text{First issue date}}$

Ref $RPI_{Div\ payment\ date}$ = $RPI_{October\ 2004}$ + (25/31) [RPI $_{November\ 2004}$ – $RPI_{October2004}$]

= 208.3 + (25/31)[209.9-208.3)
= 209.59032 (Rounded to 5 decimal places)

The Ref RPI was calculated in previous example as 202.40323.

The index ratio is 209.59032/202.40323 = 1.03551 (Rounded to 5 decimal places)

Semi-annual coupon payment = 4/2 x 1.03551
= £2.071020 (Rounded to 6 decimal places).

Assuming no future inflation, an RPI value of 205.0 will be used for both October 2004 and November 2004. The calculation is shown below.

Ref $RPI_{Div\ payment\ date}$ = 205.0 + (25/31)[205.0-205.0] = 205.00000

The index Ratio = 205.00000/202.40323 = 1.01283 (Rounded to 5 decimal places)

Semi-annual coupon payment = 4/2 x 1.01283 = £2.025660 (Rounded to 6 decimal places).

7.5 Accrued Interest Calculations

Interest accrues on an actual/actual basis and is calculated to the settlement date.

$$\frac{tc}{s} \times \frac{C}{2} \times \text{Index Ratio}_{\text{Settlement date}}$$

Where:

tc is the number of calendar days from the previous quasi-coupon date to the settlement date

s is the number of calendar days in the quasi-coupon period

C is the annual coupon.

The Index $Ratio_{Settlement\ date}$ = Ref $RPI_{Settlement\ date}$/Ref $RPI_{First\ issue\ date}$

$$\text{Ref RPI}_{\text{Settlement date}} = \text{Ref RPI}_M + \left(\frac{t-1}{D}\right)[\text{Ref RPI}_{M+1} - \text{Ref RPI}_M]$$

Where:

D = Number of days in calendar month in which the settlement date falls.

t = Calendar days corresponding to the given date.

RPI_M = Reference RPI for the first day of the calendar month in which the settlement date falls.

Ref RPI_{M+1} = Reference RPI for the first day of the calendar month immediately following the settlement date.

Assuming settlement date of 24th June 2004 and RPI of 201.5 for March 2004 and 203.0 for April 2004. Issue date of 26th January 2004

Ref $RPI_{Settlement\ date} = RPI_{March\ 2004} + (23/30)\ [RPI_{April\ 2004} - RPI_{March\ 2004}]$

$= 201.5 + (23/30)\ [203.0 - 201.5]$

= 202.64999 (Rounded to 5 decimal places)

The Ref RPI was calculated in previous example as 202.40323

Index $Ratio_{Settlement\ date} = 202.64999/202.40323 = 1.00122$ (Rounded to 5 decimal places)

Accrued interest = 150/182 x 4/2 x 1.00122 = £1.350363 (Rounded to 6 decimal places)

APPENDIX A

ACCRUED INTEREST CALCULATIONS

Both conventional and index-linked gilts accrue interest daily on an actual/actual basis. The accrued interest, when purchasing a gilt, is calculated to the settlement date.

The basic principle used in the accrued interest calculation is that from one normal interest payment date to the next, the gilt accrues interest at the same amount for each calendar day at such a rate that it will have accrued exactly the payment amount by the coupon date.

Example

Consider a gilt with a 6% coupon, which pays 3% on 7 March and on 7 September.
The daily accrued interest per £100 nominal of the gilt between the 7 March and 7 September is 3/184% as there are 184 days between 7 March and 7 September.

However, the daily accrued interest between 7 September and 7 March is either 3/181% = 0.0166% (181 days in a non-leap year), or 3/182% = 0.0165% (182 days in a leap year).

Example
2½% Treasury Index-Linked Stock 2013

The 2 ½% Treasury Index-linked Stock 2013 was issued on 21 February 1985. It pays semi-annual coupons on 16 February and 16 August, and the adjusted Retail Price Index (RPI) at the issue base date is 89.2014.

The calculation of accrued interest for this gilt for settlement on 2 June 2004 is as follows.

The gilt last paid a coupon on 16 February 2004 and the next coupon is expected on 16 August 2004. This coupon will be based on a RPI of 183.5 (i.e. the RPI 8 months before the payment). The coupon payment is adjusted by the ratio of the two RPIs. Hence the expected payment is:

$$1.25 \times 183.5 / 89.2014 = 2.571428$$

As there are 107 days from 16 February 2004 to 2 June 2004 and 182 days from 16 February 2004 to 16 August 2004, the accrued interest on 2nd June is:

$$2.571428 \times 107 / 182 = 1.51177\%$$

Sometimes gilts have either long or short coupon payment periods. When this occurs, "pseudo" or quasi-coupon payment dates are created so that the gilt accrues interest at the same daily rate as a gilt with the same coupon with the same normal interest payment dates.

Example

Consider a gilt with a 6% coupon, which will normally pay 3% interest on 7 March and on 7 September, which has been issued this year on 7 June.

Prior to the first coupon payment on 7 September, the gilt will accrue interest at the same daily rate as a gilt with a 6% coupon which last paid interest on 7 March, i.e. at a daily rate of 3/184% (there being 184 days from 7 March to 7 September). The first coupon payment for such a gilt will be

3 x 92 / 184% = 1.5%, as there are 92 days from 7 June to 7 September.

The position in a gilt that has a long first coupon period is slightly more complicated.

APPENDIX A

Example
Consider a 6% conventional gilt, which will normally pay 3% interest on 7 March and on 7 September, which has been issued this year (not a leap year) on 7 February. It has been specified that the first coupon payment will be on 7 September, i.e. a period of 7 months.
From the 7 February to 7 March the gilt will accrue interest at daily rate of 3/181%, as there are 181 days from the previous 7 September to the 7 March. From 7 March to 7 September it will then accrue at a daily rate of 3/184%.
The first interest payment on 7 September will be (3 x 28 / 181 + 3)% = 3.4641%.

Gilts (other than the 3½% War Loan) go ex-dividend (XD) 7 business days before the coupon payment date. This means that the seller as opposed to the buyer is entitled to the accrued interest associated with the gilt, when the settlement date is in the XD period. The accrued interest for such a transaction is now **negative**. 3½% War Loan goes XD 10 business days before its coupon dates on 1 June and 1 December.

Example
Consider a 6% conventional gilt, which pays 3% on 7 March and on 7 September. It has been sold for settlement on 1 September, i.e. within the XD period, with a negative accrued interest.
The accrued interest per £100 nominal associated with the transaction is now -3 x 6 / 184% = -0.0978%, as the settlement date is 6 days before the coupon date in a 184 day period.

APPENDIX B

REDEMPTION YIELD COMPOUNDING FREQUENCY ADJUSTMENTS

In some markets it is customary to calculate redemption yields with the interest being compounded annually, whereas in others it is compounded semi-annually. This appendix shows how it is possible to convert yields from one compounding basis to another.

Redemption yields for both conventional and index-linked gilts and sectors are calculated with interest being compounded on a semi-annual basis. This is historical because nearly all the gilts pay interest twice a year. It is normal to compound interest on a semi-annual basis in markets, such as USA and Canada, where coupons are normally paid semi-annually. However, in markets where coupons are normally paid annually, such as in continental Europe, yields are normally compounded annually.

Redemption yields compounded semi-annually appear to be less than those compounded annually. This is because the holder has use of half the annual coupon for an extra six months.

Example

Consider a 10% conventional gilt, which pays annually. If it is priced at par (100) and has exactly 10 years to redemption, then it has a redemption yield compounded annually of 10%.

However, if on the other hand it were to pay 5% interest every 6 months, in 6 months time one would get £5 for every £100 of nominal held which one could invest for the next extra 6 months at 10% p.a., assuming yields do not change. Thus on an annual compounding basis the redemption yield would now be:

100 x 10 / 100 + 5 x 10 / (2 x 100) = 10.25%

Generally the formula for converting semi-annually compounded yields to annually compounded ones and vice versa is:

$$(1 + y_a) = (1 + \frac{y_s}{2})^2$$

where: y_a = yield compounded annually
y_s = yield compounded semi-annually.